

Exo 2 suite

Vous devez trouver $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

$$\frac{\partial}{\partial x} \ln(y^2+1) = \ln(y^2+1)$$

$$\frac{\partial}{\partial y} \ln(y^2+1) = \frac{\partial}{\partial y} \ln(y^2+1)$$

$$\frac{\partial}{\partial y} \frac{\partial}{\partial x} f_1 = \frac{\partial}{\partial y} \ln(y^2+1) = \frac{2y}{y^2+1}$$

$$\frac{\partial}{\partial x} \frac{\partial}{\partial y} f_1 = \frac{\partial}{\partial x} \frac{2xy}{y^2+1} = \frac{2y}{y^2+1} \frac{\partial}{\partial x} x$$

$$\frac{\partial}{\partial x} \frac{x^2}{x+2y} = \frac{x^2 + 4xy}{(x+2y)^2}$$

$$\frac{\partial}{\partial y} \frac{x^2}{x+2y} = \frac{-2x^2}{(x+2y)^2}$$

$$\frac{\partial}{\partial y} \frac{x^2 + 4xy}{(x+2y)^2} = \frac{(x+2y)^2 \times 4x - (x^2 + 4xy) \times 4y(x+2y)}{(x+2y)^4}$$

$$= \frac{(x+2y) \times 4x - (x^2 + 4xy) \times 4y}{(x+2y)^3}$$

= vous devez finir

$$\frac{\partial}{\partial x} \frac{-2x^2}{(x+2y)^2} = -2 \frac{(x+2y)^2 \times 2x - 2(x+2y)x^2}{(x+2y)^4}$$

$$= -2 \frac{(x+2y) \times 2x - 2x^2}{(x+2y)^3} = \frac{-8xy}{(x+2y)^3}$$

Exo 3

$$\begin{aligned} \frac{\partial}{\partial x} \frac{xy(x^2 - y^2)}{x^2 + y^2} &= y \frac{\partial}{\partial x} \frac{x^3}{x^2 + y^2} + y^3 \frac{\partial}{\partial x} \frac{x}{x^2 + y^2} \\ &= y \frac{(x^4 + 4x^2y^2 - y^4)}{(x^2 + y^2)^2} \end{aligned}$$

$$\frac{\partial}{\partial y} \frac{xy(x^2 - y^2)}{x^2 + y^2} = x \frac{(x^4 - 4x^2y^2 - y^4)}{(x^2 + y^2)^2}$$

Vous pourrez chercher $\frac{\partial^2}{\partial x \partial y}$ et $\frac{\partial^2}{\partial y \partial x}$ comme d'habitude mais il y a une astuce!

$$\frac{\partial f}{\partial x}(0, t) = -\frac{t^5}{t^4} \quad \frac{\partial f}{\partial y}(t, 0) = \frac{t^5}{t^4}$$

$$\Rightarrow \frac{\partial f}{\partial y \partial x} = \frac{d}{dt} \frac{\partial f}{\partial x}(0, t) = \frac{d}{dt} -t = -1$$